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The case when A_1 is obtuse may be treated by a similar method.

Also solved by A. M. Harding, Nathan Altshiller, C. C. Yen, J. F. Lü, K. K. Chan, Horace Olson, G. Breit, H. C. Gossard, Frank V. Morley, Louis Weisner, and Otto J. Ramler.

428 (Calculus). Proposed by J. L. RILEY, Northwestern State Normal School, Tahlequah, Okla.

The loop of a lemniscate rolls in contact with the axis of x. Prove that the locus of the node is given by the equation

$$1+\left(\frac{dy}{dx}\right)^2=\left(\frac{a}{y}\right)^{4/3},$$

and that $2\rho\rho' = a^2$, if ρ, ρ' be corresponding radii of curvature of this locus and the lemniscate.

Solution by A. M. Harding, University of Arkansas.

The problem, as originally stated, is incorrect. It should read as above. The equation of the lemniscate, referred to a tangent at its center, is $r^2 = a^2 \sin 2\theta$. By the formulas of the elementary calculus it is easily shown that

$$s=\int_0^\theta a\sqrt{\csc 2\theta}\,d\theta, \qquad \psi = 2\theta, \qquad \text{and} \qquad \rho=\frac{a}{3}\sqrt{\csc 2\theta},$$

where s is the length of arc from the node $A(x, y)$ to the point P, ψ is the angle between AP and the tangent at P, and ρ is the radius of curvature at P.

Let the lemniscate roll along the x-axis until the tangent at P coincides with this axis, and let O be the origin of coördinates. Then

$$x = OP - AP\cos\psi = a\int_0^\theta \sqrt{\csc 2\theta}\,d\theta - a\sqrt{\sin 2\theta}\cos 2\theta, \text{ and } y = AP\sin\psi = a\sqrt{\sin 2\theta}\sin 2\theta;$$

whence,

$$\frac{dy}{dx} = \cot 2\theta,$$

and

$$\frac{d^2y}{dx^2} = -\frac{2(\csc 2\theta)^{3/2}}{3a\sin^2 2\theta}.$$

Hence,

$$1+\left(\frac{dy}{dx}\right)^2 = \csc^2 2\theta = \left(\frac{a}{y}\right)^{4/3}$$

and

$$\rho' = -\frac{3}{2}a\sqrt{\sin 2\theta}.$$

Hence,

$$\rho\rho' = -\frac{a^2}{2}.$$

Also solved by William Hoover.

430 (Calculus). Proposed by G. PAASWELL, New York City.

Revolve a circle about a chord (not a diameter). Select a system of rectilinear coördinates with this chord as one axis and the origin as the intersection of the chord and the circumference. Term this axis the z-axis and pass a plane through the x- (or y-) axis. Find the area of this surface intercepted by this plane and the xz- (or yz-) plane.

Solution by William Hoover, Columbus, Ohio.

Let a = the radius of the circle, c = the distance of the chord from the center, 2α = the angle subtended at the center by the arc of the chord; take the middle of the chord for the origin, and the radius at right angles to the chord for the x-axis; the equation to the generating arc is then